

SCORE: \_\_\_\_ / 35 POINTS

1. NO CALCULATORS ALLOWED
2. SHOW PROPER WORK & SIMPLIFY YOUR FINAL ANSWER TO RECEIVE FULL CREDIT

Consider the polar equation  $r = 1 + 2 \cos 2\theta$ .

SCORE: \_\_\_\_ / 13 PTS

- [a] Your partner runs the following symmetry tests, and gets the following correct results:

If you replace  $(r, \theta)$  with  $(-r, \theta)$ , you can **NOT** simplify the equation back to the original equation.

If you replace  $(r, \theta)$  with  $(-r, -\theta)$ , you can **NOT** simplify the equation back to the original equation.

If you replace  $(r, \theta)$  with  $(r, -\theta)$ , you can simplify the equation back to the original equation.

Based only on the tests above, what can you conclude about whether the graph of the equation is symmetric

[i] with respect to  $\theta = \frac{\pi}{2}$  ?

[ii] with respect to the polar axis ?

[iii] with respect to the pole ?

CAN'T TELL  $\left(\frac{1}{2}\right)$

SYMMETRIC  $\left(\frac{1}{2}\right)$

CAN'T TELL  $\left(\frac{1}{2}\right)$

- [b] Determine whether the graph is symmetric with respect to the pole, the polar axis, and  $\theta = \frac{\pi}{2}$ .

You may use any or all of the test results in part [a] without rerunning the tests.

POLE  $(r, \pi + \theta)$ :  $r = 1 + 2 \cos 2(\pi + \theta)$   $\left(\frac{1}{2}\right)$   
 $= 1 + 2 \cos(2\pi + 2\theta)$   $\left(\frac{1}{2}\right)$   
 $= 1 + 2 [\cos 2\pi \cos 2\theta - \sin 2\pi \sin 2\theta]$   $\left(\frac{1}{2}\right)$   
 $= 1 + 2 \cos 2\theta$   $\left(\frac{1}{2}\right)$  SYMMETRIC OVER POLE, POLAR  
AXIS +  $\theta = \frac{\pi}{2}$   $\left(\frac{1}{2}\right)$

- [c] What is the minimum interval for  $\theta$  that you would need to plot points before using symmetry to finish drawing the graph ?

$[0, \frac{\pi}{2}]$   $\left(\frac{1}{2}\right)$

- [d] Find the zeros of the polar equation in the minimum interval from part [c].

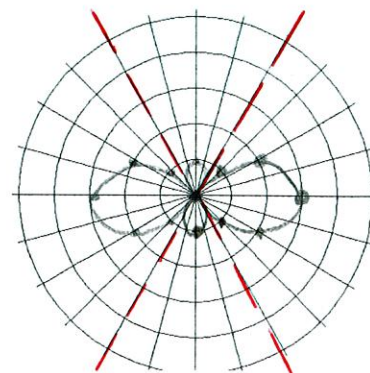
$\left(\frac{1}{2}\right)$   $0 = 1 + 2 \cos 2\theta$   $0 \leq \theta \leq \frac{\pi}{2}$   
 $\left(\frac{1}{2}\right)$   $\cos 2\theta = -\frac{1}{2}$   $0 \leq 2\theta \leq \pi$   
 $2\theta = \frac{2\pi}{3}$   
 $\left(\frac{1}{2}\right)$   $\theta = \frac{\pi}{3}$

- [e] Graph the polar equation using symmetry, zeros and any other additional points.

**NOTE: You must find the value of  $r$  for all common angles from the minimum interval.**

$\theta$	$r$
0	$1 + 2 \cos 2(0) = 3$ $\left(\frac{1}{2}\right)$
$\frac{\pi}{6}$	$1 + 2 \cos 2(\frac{\pi}{6}) = 2$ $\left(\frac{1}{2}\right)$
$\frac{\pi}{4}$	$1 + 2 \cos 2(\frac{\pi}{4}) = 1$ $\left(\frac{1}{2}\right)$
$\frac{\pi}{3}$	$1 + 2 \cos 2(\frac{\pi}{3}) = 0$
$\frac{\pi}{2}$	$1 + 2 \cos 2(\frac{\pi}{2}) = -1$ $\left(\frac{1}{2}\right)$

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ME





**1. NO CALCULATORS ALLOWED**  
**2. SHOW PROPER WORK & SIMPLIFY YOUR FINAL ANSWER TO RECEIVE FULL CREDIT**

Consider the polar equation  $r = \frac{5}{3 + 2\cos\theta}$ .

SCORE: \_\_\_\_ / 9 PTS

- [a] Find the rectangular coordinates of all intercepts of the graph. NOTE: Do NOT convert the equation to rectangular form.

$\theta$	$r$	
0	1	$\textcircled{1} (1, 0) \textcircled{1} (-5, 0) \textcircled{1}$
$\frac{\pi}{2}$	$\frac{5}{3}$	$\textcircled{1} (0, \frac{5}{3}) \textcircled{1} (0, -\frac{5}{3}) \textcircled{1}$
$\pi$	$\frac{5}{3}$	
$\frac{3\pi}{2}$	$\frac{5}{3}$	

- [b] Find the rectangular coordinates of all latera recta of the graph. NOTE: "Latera recta" is the plural of "latus rectum".  
You will need to find the rectangular coordinates of the foci first.

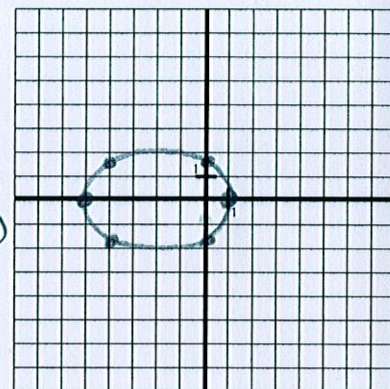
CENTER =  $(\frac{1+5}{2}, 0) = (-2, 0)$

FOCUS =  $(2(-2), 0) = (-4, 0)$

LR =  $\textcircled{\frac{1}{2}} (-4, \frac{5}{3}) \textcircled{\frac{1}{2}} (-4, -\frac{5}{3}) \textcircled{\frac{1}{2}} (0, \frac{5}{3}) \textcircled{\frac{1}{2}} (0, -\frac{5}{3})$

- [c] Graph the polar equation using all points from parts [a] and [b].

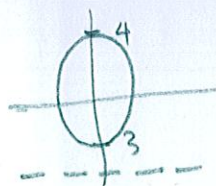
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Find the polar equation of the ellipse with foci at the pole and vertices at polar coordinates  $(4, \frac{\pi}{2})$  and  $(3, \frac{3\pi}{2})$ .

SCORE: \_\_\_\_ / 7 PTS

Simplify your final answer.



$r = \frac{ep}{1 - e\sin\theta} \textcircled{1}$

$= \frac{\frac{1}{7}(24)}{1 - \frac{1}{7}\sin\theta} \cdot \frac{7}{7}$

$= \frac{24}{7 - \sin\theta} \textcircled{1}$

$4 = \frac{ep}{1 - e} \textcircled{1} \quad 3 = \frac{ep}{1 + e} \textcircled{1}$

$ep = 4(1 - e) \quad ep = 3(1 + e)$

$4 - 4e = 3 + 3e \textcircled{1}$

$1 = 7e$

$e = \frac{1}{7} \textcircled{1} \quad x = \frac{1}{t - 2}$

$\frac{1}{7}p = 3(\frac{8}{7})$

$p = 24 \textcircled{1}$

Eliminate the parameter to find the rectangular equation corresponding to the parametric equations

SCORE: \_\_\_\_ / 6 PTS

Simplify your final answer.

$x(t - 2) = 1$   
 $t - 2 = \frac{1}{x}$   
 $t = \frac{1}{x} + 2$   
 $\textcircled{2}$

$y = \frac{\frac{1}{x} + 2}{\frac{1}{x} + 3} \cdot \frac{x}{x}$   
 $\textcircled{2}$

$y = \frac{1 + 2x}{1 + 3x} \textcircled{2}$